Experiences with using R in credit risk

Hong Ooi

ANZ

Introduction

Not very sexy....

- LGD haircut modelling
- Through-the-cycle calibration
- Stress testing simulation app
- SAS and R
- Closing comments

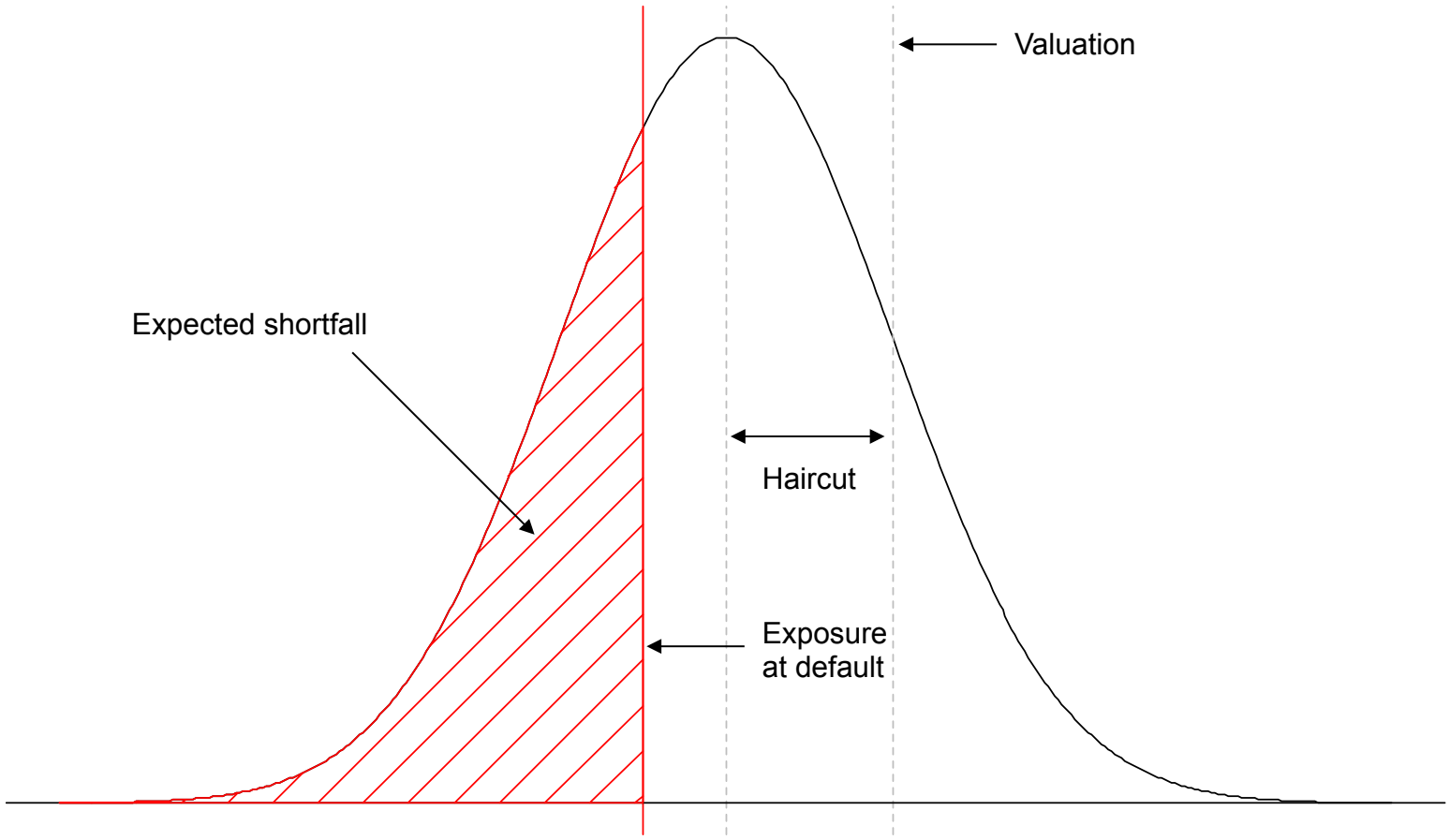
Mortgage haircut model

- When a mortgage defaults, the bank can take possession of the property and sell it to recoup the loss[1]
- We have some idea of the market value of the property
- Actual sale price tends to be lower on average than the market value (the *haircut*)[2]
- If sale price > exposure at default, we don't make a loss (excess is passed on to customer); otherwise, we make a loss

Expected loss = P(default) x EAD x P(possess) x exp.shortfall

Notes:

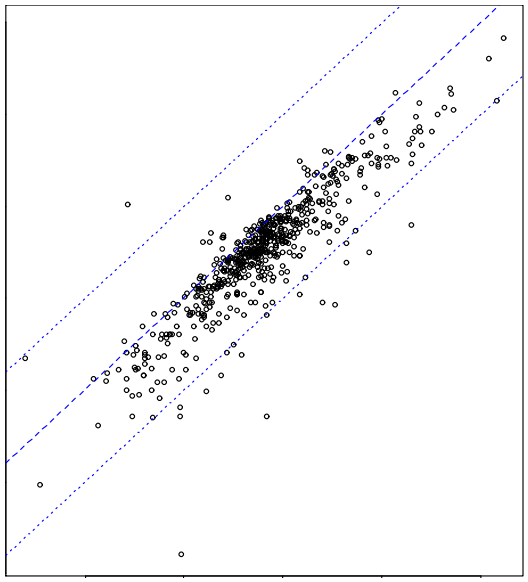
1. For ANZ, <10% of defaults actually result in possession
2. Meaning of "haircut" depends on context; very different when talking about, say, US mortgages


Valuation
Expected shortfall
Haircut
Exposure
at default

Stat modelling

- Modelling part is in finding parameters for the sale price distribution
  - Assumed distributional shape, eg Gaussian
  - Mean haircut relates average sale price to valuation
  - Spread (volatility) of sale price around haircut
- Once model is found, calculating expected shortfall is just (complicated) arithmetic

Valuation at origination



Valuation at kerbside

Valuation after possession

Volatility

Volatility of haircut (=sale price/valuation) appears to vary systematically:

Property type	SD(haircut)*
A	11.6%
B	9.3%
C	31.2%

State/territory	SD(haircut)*
1	NA
2	13.3%
3	7.7%
4	9.2%
5	15.6%
6	18.4%
7	14.8%

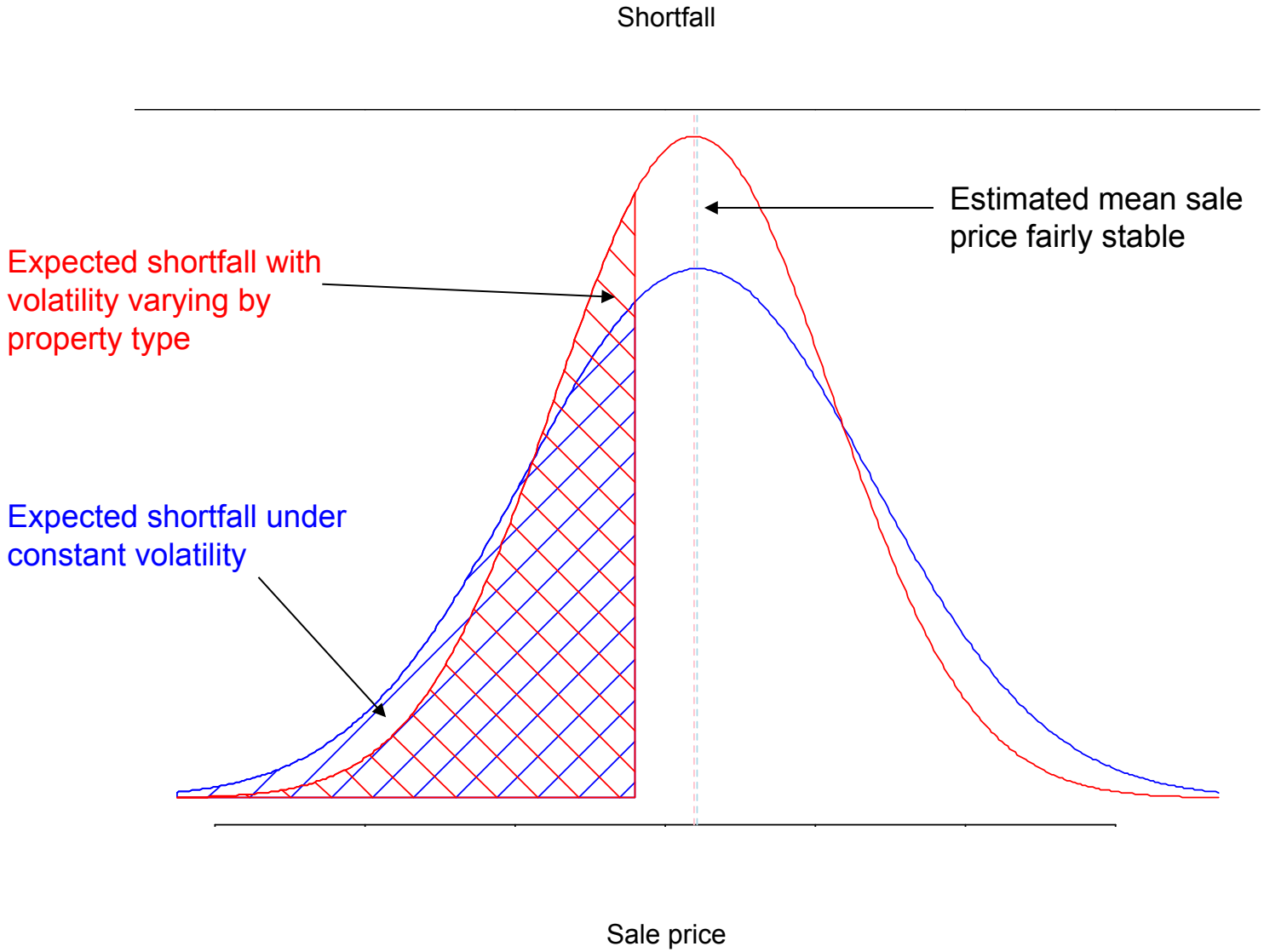
* valued after possession

Volatility modelling

- Use regression to estimate haircut as a function of loan characteristics
- Hierarchy of models, by complexity:
  - Constant volatility
  - Volatility varying by property type
  - Volatility varying by property type and state
- Use log-linear structure for volatility to ensure +ve variance estimates

- Constant volatility model is ordinary linear regression
- Varying-volatility models can be fit by generalised least squares, using `gls` in the `nlme` package
  - Simpler and faster to directly maximise the Gaussian likelihood with `optim`/`nlminb` (latter will reproduce `gls` fit)

ANZ


Shortfall
Estimated mean sale price fairly stable
Expected shortfall with volatility varying by property type
Expected shortfall under constant volatility
Sale price

Volatility: T-regression

- Still assumes Gaussian distribution for volatility
- Data can be more heavy-tailed than the Gaussian, even after deleting outliers
  - Inflates estimates of variance

- Solution: replace Gaussian distribution with *t* distribution on small df
  - df acts as a shape parameter, controls how much influence outlying observations have
  - Coding this is a straightforward extension of the Gaussian: simply change all `*norm` functions to `*t`
    - Can still obtain Gaussian model by setting `df=Inf`
  - cf Andrew Robinson's previous MelbURN talk on robust regression and ML fitting

ANZ

Example impact

Property type = C, state = 7, valuation $250k, EAD $240k

Gaussian model

Mean formula	Volatility formula	Expected shortfall ($)
~1	~1	7,610
~1	~proptype	23,686
~1	~proptype + state	29,931

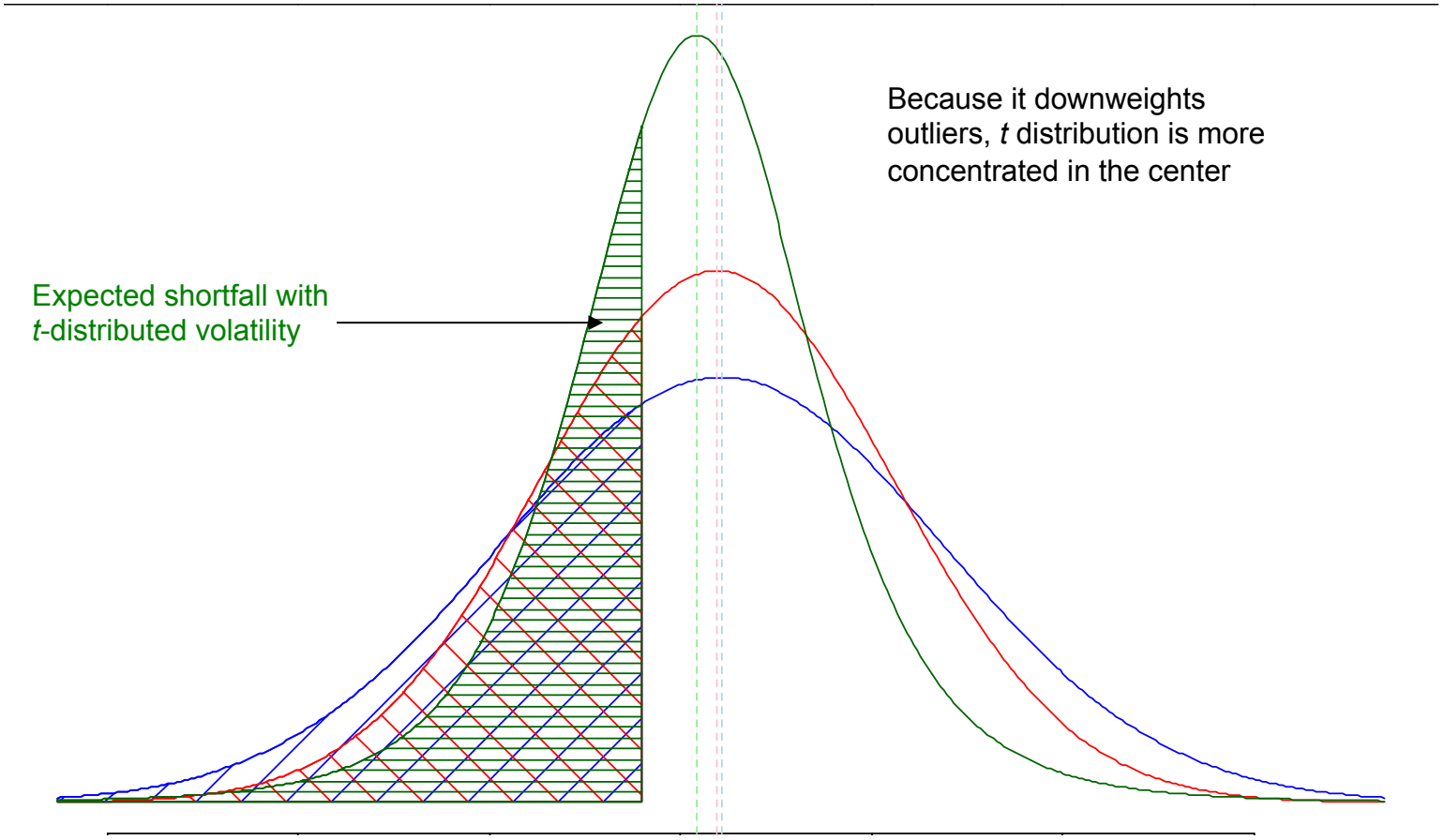
t_5 model

Mean formula	Volatility formula	Expected shortfall ($)
~1	~1	4,493
~1	~proptype	10,190
~1	~proptype + state	5,896

Model fitting function (simplified)

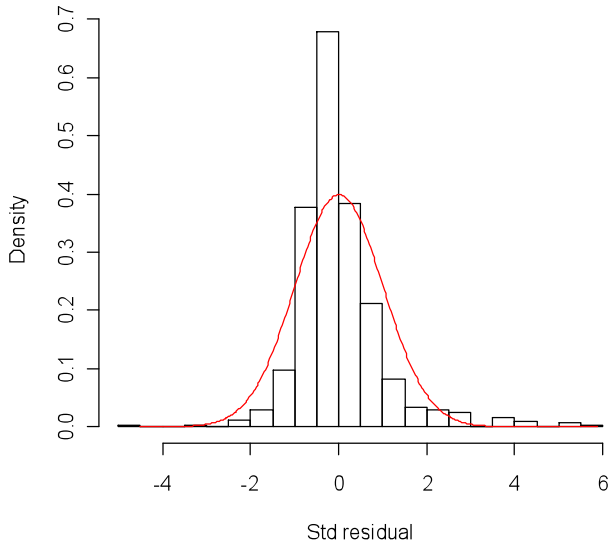
```
tmod <- function(formula, formula.s, data, df, ...)
{
    mf <- mf.s <- match.call(expand.dots = FALSE)
    mf.s[["formula"]] <- mf.s$formula.s
    mf$df <- mf$formula.s <- mf$... <- mf.s$df <- mf.s$formula.s <- mf.s$... <- NULL
    mf[[1]] <- mf.s[[1]] <- as.name("model.frame")
    mf <- eval(mf, parent.frame())
    mf.s <- eval(mf.s, parent.frame())
    mm <- model.matrix(attr(mf, "terms"), mf)
    mm.s <- model.matrix(attr(mf.s, "terms"), mf.s)
    y <- model.response(mf)
    p <- ncol(mm)
    p.s <- ncol(mm.s)
    t.nll <- function(par, y, Xm, Xs)
    {
        m <- Xm %*% par[1:p]
        logs <- Xs %*% par[-(1:p)]
        -sum(dt((y - m)/exp(logs), df = df, log = TRUE) - logs)
    }
    lmf <- lm.fit(mm, y) # use ordinary least-squares fit as starting point
    par <- c(lmf$coefficients, log(sd(lmf$residuals)), rep(0, p.s - 1))
    names(par) <- c(colnames(mm), colnames(mm.s))
    nlminb(par, t.nll, y = y, Xm = mm, Xs = mm.s, ...)
}

tmod(salepr/valuation ~ 1, ~ proptype + state, data = haircut, df = Inf)
tmod(salepr/valuation ~ proptype, ~ proptype, data = haircut, df = 5)
```

Shortfall

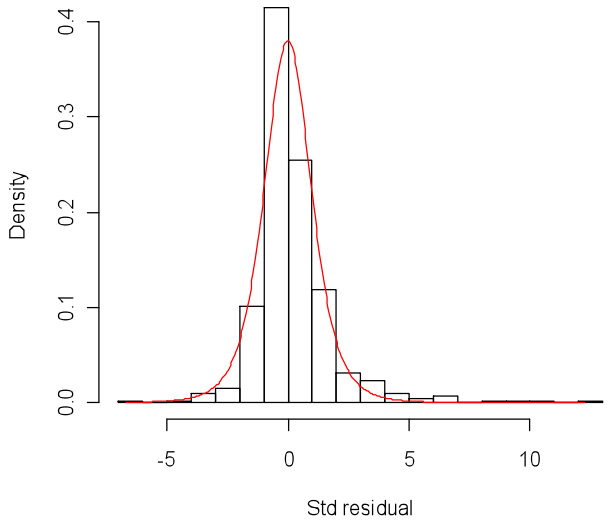


Sale price

Normal model residuals



t_5-model residuals



Notes on model behaviour

- Why does using a heavy-tailed error distribution reduce the expected shortfall?
  - With normal distrib, volatility is overestimated
    - → Likelihood of low sale price is also inflated
    - *t* distrib corrects this
  - Extreme tails of the *t* less important
    - At lower end, sale price cannot go below 0
    - At higher end, sale price > EAD is gravy
  - This is not a monotonic relationship! At low enough thresholds, eventually heavier tail of the *t* will make itself felt

- In most regression situations, assuming sufficient data, distributional assumptions (ie, normality, homoskedasticity) are not so critical: CLT comes into play
- Here, they are important: changing the distributional assumptions can change expected shortfall by big amounts

In SAS

- SAS has PROC MIXED for modelling variances, but only allows one grouping variable and assumes a normal distribution
- PROC NLIN does general nonlinear optimisation
- Also possible in PROC IML

- None of these are as flexible or powerful as R
- The R modelling function returns an *object*, which can be used to generate predictions, compute summaries, etc
- SAS 9.2 now has PROC PLM that does something similar, but requires the modelling proc to execute a STORE statement first
  - Only a few procs support this currently
  - If you're fitting a custom model (like this one), you're out of luck

Through-the-cycle calibration

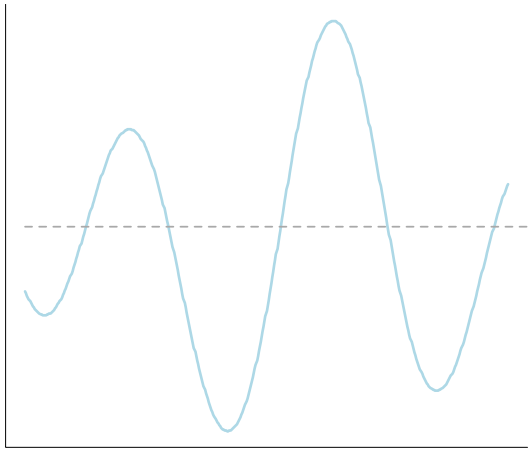
- For capital purposes, we would like an estimate of default probability that doesn't depend on the current state of the economy
- This is called a *through-the-cycle* or *long-run* PD
  - Contrast with a *point-in-time* or *spot* PD, which is what most models will give you (data is inherently point-in-time)
- Exactly what long-run means can be the subject of philosophical debate; I'll define it as a customer's average risk, given their characteristics, across the different economic conditions that might arise
  - This is *not* a lifetime estimate: eg their age/time on books doesn't change
  - Which variables are considered to be cyclical can be a tricky decision to make (many behavioural variables eg credit card balance are probably correlated with the economy)
  - During bad economic times, the long-run PD will be below the spot, and vice-versa during good times
    - You don't want to have to raise capital during a crisis

TTC approach

- Start with the spot estimate:

$$\mathrm{PD}(x, e) = f(x, e)$$

- x = individual customer's characteristics
- e = economic variables (constant for all customers at any point in time)
- Average over the possible values of e to get a TTC estimate



TTC approach

- This is complicated numerically, can be done in various ways eg Monte Carlo
- Use backcasting for simplicity: take historical values of *e*, substitute into prediction equation, average the results
  - As we are interested in means rather than quantiles, this shouldn't affect accuracy much (other practical issues will have much more impact)
- R used to handle backcasting, combining multiple spot PDs into one output value

TTC calculation

- Input from spot model is a prediction equation, along with sample of historical economic data

```
spot_predict <- function(data)
{
  # code copied from SAS; with preprocessing, can be arbitrarily complex
  xb <- with(data, b0 + x1 * b1 + x2 * b2 + ... )
  plogis(xb)
}

ttc_predict <- function(data, ecodata, from = "2000-01-01", to = "2010-12-01")
{
  dates <- seq(as.Date(from), as.Date(to), by = "months")
  evars <- names(ecodata)
  pd <- matrix(nrow(data), length(dates))
  for(i in seq_along(dates))
  {
    data[evars] <- subset(ecodata, date == dates[i], evars)
    pd[, i] <- spot_predict(data)
  }
  apply(pd, 1, mean)
}
```

Binning/cohorting

- Raw TTC estimate is a combination of many spot PDs, each of which is from a logistic regression

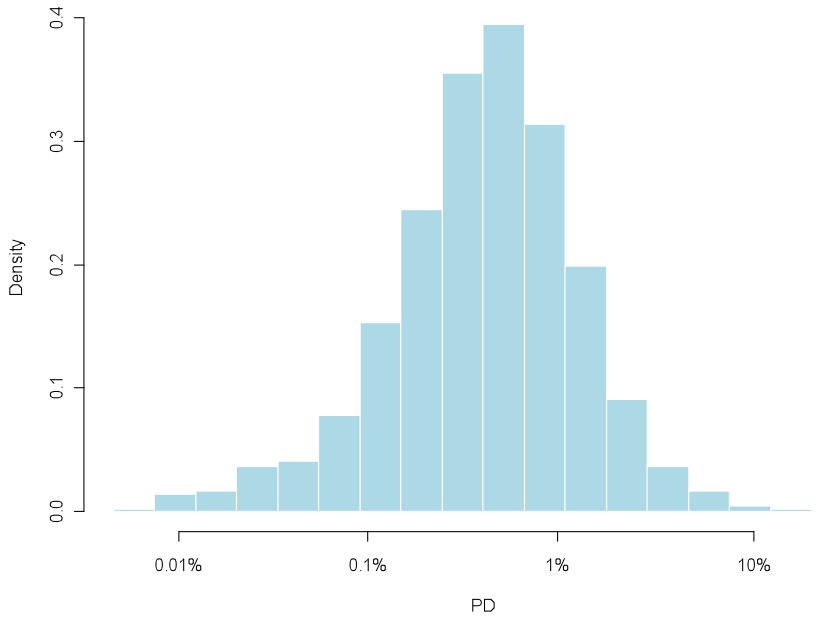
  → TTC estimate is a complicated function of customer attributes
- Need to simplify for communication, implementation purposes

- Turn into *bins* or *cohorts* based on customer attributes: estimate for each cohort is the average for customers within the cohort
- Take pragmatic approach to defining cohorts
  - Create tiers based on small selection of variables that will split out riskiest customers
  - Within each tier, create contingency table using attributes deemed most interesting/important to the business
  - Number of cohorts limited by need for simplicity/manageability, <1000 desirable
  - Not a data-driven approach, although selection of variables informed by data exploration/analysis

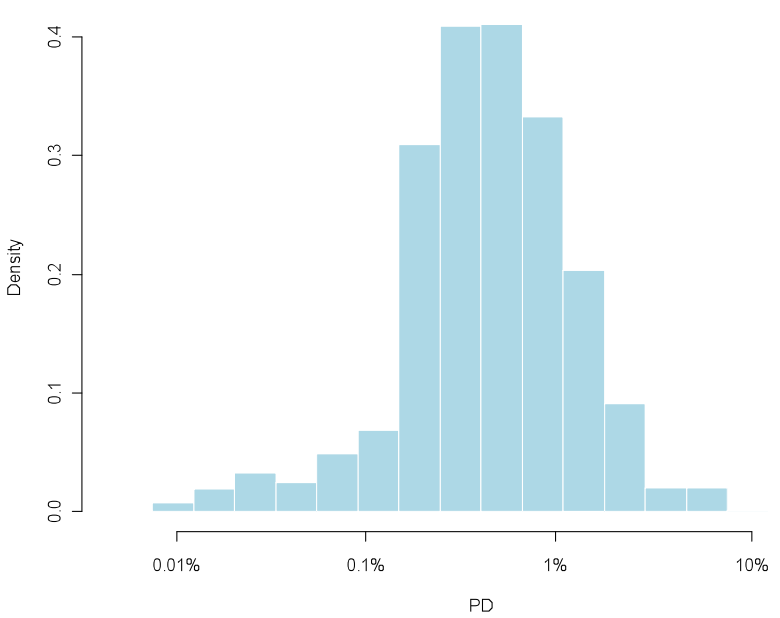
Binning/cohorting

Example from nameless portfolio:

Raw TTC PD



Cohorted TTC PD



Binning input

```
varlist <- list(
    low_doc2=list(name="low_doc",
                  breaks=c("N", "Y"),
                  midp=c("N", "Y"),
                  na.val="N"),
    enq     =list(name="wrst_nbr_enq",
                  breaks=c(-Inf, 0, 5, 15, Inf),
                  midp=c(0, 3, 10, 25),
                  na.val=0),
    lvr     =list(name="new_lvr_basel",
                  breaks=c(-Inf, 60, 70, 80, 90, Inf),
                  midp=c(50, 60, 70, 80, 95),
                  na.val=70),
    ...
```

by application of `expand.grid()`...

	low_doc	wrst_nbr_enq	new_lvr_basel	...	tier1	tier2
1	N	0	50	...	1	1
2	N	0	60	...	1	2
3	N	0	70	...	1	3
4	N	0	80	...	1	4
5	N	0	95	...	1	5
6	N	3	50	...	1	6
7	N	3	60	...	1	7
8	N	3	70	...	1	8
9	N	3	80	...	1	9
10	N	3	95	...	1	10

Binning output

```
if low_doc = ' ' then low_doc2 = 1;
else if low_doc = 'Y' then low_doc2 = 1;
else low_doc2 = 2;

if wrst_nbr_enq = . then enq = 0;
else if wrst_nbr_enq <= 0 then enq = 0;
else if wrst_nbr_enq <= 5 then enq = 3;
else if wrst_nbr_enq <= 15 then enq = 10;
else enq = 25;

if new_lvr_basel = . then lvr = 70;
else if new_lvr_basel <= 60 then lvr = 50;
else if new_lvr_basel <= 70 then lvr = 60;
else if new_lvr_basel <= 80 then lvr = 70;
else if new_lvr_basel <= 90 then lvr = 80;
else lvr = 95;
...
```

```
if lvr = 50 and enq = 0 and low_doc = 'N' then do; tier2 = 1; ttc_pd = _________; end;
else if lvr = 60 and enq = 0 and low_doc = 'N' then do; tier2 = 2; ttc_pd = _________; end;
else if lvr = 70 and enq = 0 and low_doc = 'N' then do; tier2 = 3; ttc_pd = _________; end;
else if lvr = 80 and enq = 0 and low_doc = 'N' then do; tier2 = 4; ttc_pd = _________; end;
else if lvr = 95 and enq = 0 and low_doc = 'N' then do; tier2 = 5; ttc_pd = _________; end;
else if lvr = 50 and enq = 3 and low_doc = 'N' then do; tier2 = 6; ttc_pd = _________; end;
else if lvr = 60 and enq = 3 and low_doc = 'N' then do; tier2 = 7; ttc_pd = _________; end;
...
```

Binning/cohorting

R code to generate SAS code for scoring a dataset:

```
sas_all <- character()
for(i in seq_along(tierdefs))
{
    tvars <- tierdefs[[i]]
    varnames <- lapply(varlist[tvars], `[[`, "name")
    this_tier <- which(celltable$tier == i)
    sas <- "if"
    for(j in seq_along(tvars))
    {
        sas <- paste(sas, tvars[j], "=", as.numeric(celltable[this_tier, varnames[j]]))
        if(j < length(tvars))
            sas <- paste(sas, "and")
    }
    sas <- paste(sas, sprintf("then do; tier2 = %d; ttc_pd = %s;", celltable$tier2[this_tier],
             celltable$ttc_pd[this_tier]))
    sas[-1] <- paste("else", sas[-1])
    sas_all <- c(sas_all, sas,
        sprintf("else put 'ERROR: unhandled case, tier = %d  n = ' _n_;", i))
}
writeLines(sas_all, sasfile)
```

Stress testing simulation

- Banks run stress tests on their loan portfolios, to see what a downturn would do to their financial health
- Mathematical framework is similar to the "Vasicek model":
  - Represent the economy by a parameter X
  - Each loan has a transition matrix that is shifted based on X, determining its risk grade in year t given its grade in year t - 1
    - Defaults if bottom grade reached
- Take a scenario/simulation-based approach: set X to a stressed value, run N times, take the average
  - Contrast to VaR: "average result for a stressed economy", as opposed to "stressed result for an average economy"
- Example data: portfolio of 100,000 commercial loans along with current risk grade, split by subportfolio
- Simulation horizon: ~3 years

Application outline

- Front end in Excel (because the business world lives in Excel)
  - Calls SAS to setup datasets
    - Calls R to do the actual computations

- Previous version was an ad-hoc script written entirely in SAS, took ~4 hours to run, often crashed due to lack of disk space
  - Series of DATA steps (disk-bound)
  - Transition matrices represented by unrolled if-then-else statements (25x25 matrix becomes 625 lines of code)
- Reduced to 2 minutes with R, 1 megabyte of code cut to 10k
  - No rocket science involved: simply due to using a better tool
  - Similar times achievable with PROC IML, of which more later

ANZ

Application outline

- For each subportfolio and year, get the *median result* and store it
- Next year's simulation uses this year's median portfolio
- To avoid having to store multiple transited copies of the portfolio, we manipulate random seeds

```
for(p in 1:nPortfolios) # varies by project
{
  for(y in 1:nYears)    # usually 2-5
  {
    seed <- .GlobalEnv$.Random.seed
    for(i in 1:nIters)  # around 1,000, but could be less
      result[i] = summary(doTransit(portfolio[p, y], T[p, y]))

    med = which(result == median(result))
    portfolio[p, y + 1] = doTransit(portfolio[p, y], T[p, y], seed, med)
  }
}
```

Data structures

- For business reasons, we want to split the simulation by subportfolio
- And also present results for each year separately

  → Naturally have 2-dimensional (matrix) structure for output: [*i*, *j*]th entry is the result for the *i*th subportfolio, *j*th year

- But desired output for each [*i*, *j*] might be a bunch of summary statistics, diagnostics, etc

  → Output needs to be a list

- Similarly, we have a separate input transition matrix for each subportfolio and year

  → Input should be a matrix of matrices

Data structures

R allows matrices whose elements are lists:

```
T <- matrix(list(), nPortfolios, nYears)
for(i in 1:nPortfolios) for(j in 1:nYears)
    T[[i, j]] <- getMatrix(i, j, ...)

M <- matrix(list(), nPortfolios, nYears)
M[[i, j]]$result <- doTransit(i, j, ...)
M[[i, j]]$sumstat <- summary(M[[i, j]]$result)
```

Data structures

- Better than the alternatives:
  - List of lists: `L[[i]][[j]]` contains data that would be in `M[[i, j]]`
    - Lose ability to operate on/extract all values for a given *i* or *j* via matrix indexing
  - Separate matrices for each statistic of interest (ie, doing it Fortran-style)
    - Many more variables to manage, coding becomes a chore
    - No structure, so loops may involve munging of variable names
  - Multidimensional arrays conflate data and metadata

- But can lead to rather cryptic code:

```
getComponent <- function(x, component)
{
    x[] <- lapply(x, `[[`, component)
    lapply(apply(x, 1, I), function(xi) do.call("rbind", xi))
}
getComponent(M, "sumstat")
```


ANZ

PROC IML: a gateway to R

- As of SAS 9.2, you can use IML to execute R code, and transfer datasets to and from R:

```
PROC IML;
  call ExportDataSetToR('portfol', 'portfol');  /* creates a data frame */
  call ExportMatrixToR("&Rfuncs", 'rfuncs');
  call ExportMatrixToR("&Rscript", 'rscript');
  call ExportMatrixToR("&nIters", 'nIters');
  ...
  submit /R;
    source(rfuncs)
    source(rscript)
  endsubmit;
  call ImportDataSetFromR('result', 'result');
QUIT;
```

- No messing around with import/export via CSV, transport files, etc
- Half the code in an earlier version was for import/export

IML: a side-rant

- IML lacks:
  - Logical vectors: everything has to be numeric or character
  - Support for zero-length vectors (you don't realise how useful they are until they're gone)
  - Unoriented vectors: everything is either a row or column vector (technically, everything is a matrix)
- So something like `x = x + y[z < 0];` fails in three ways

- IML also lacks anything like a dataset/data frame: everything is a matrix
  → It's easier to transfer a SAS dataset to and from R, than IML

- *Everything is a matrix*: no lists, let alone lists of lists, or matrices of lists
  - Not even multi-way arrays

- Which puts the occasional online grouching about R into perspective

Other SAS/R interfaces

- SAS has a proprietary dataset format (or, many proprietary dataset formats)
  - R's `foreign` package includes `read.ssd` and `read.xport` for importing, and `write.foreign(*, package="SAS")` for exporting
  - Package `Hmisc` has `sas.get`
  - Package `sas7bdat` has an experimental reader for this format
  - Revolution R can read SAS datasets
  - All have glitches, are not widely available, or not fully functional
    - First 2 also need SAS installed
- SAS 9.2 and IML make these issues moot
  - You just have to pay for it
  - Caveat: only works with R <= 2.11.1 (2.12 changed the locations of binaries)
    - SAS 9.3 will support R 2.12+

ANZ

R and SAS rundown

- Advantages of R
  - Free! (base distribution, anyway)
  - Very powerful statistical programming environment: SAS takes 3 languages to do what R does with 1
  - Flexible and extensible
  - Lots of features (if you can find them)
    - User-contributed packages are a blessing and a curse
  - Ability to handle large datasets is improving
- Advantages of SAS
  - Pervasive presence in large firms
    - "Nobody got fired for buying ~~IBM~~ SAS"
    - Compatibility with existing processes/metadata
    - Long-term support
  - Tremendous data processing/data warehousing capability
  - Lots of features (if you can afford them)
  - Sometimes cleaner than R, especially for data manipulation

R and SAS rundown

Example: get weighted summary statistics by groups

```
proc summary data = indat nway;
  class a b;
  var x y;
  weight w;
  output sum(w)=sumwt mean(x)=xmean mean(y)=ymean var(y)=yvar
    out = outdat;
run;
```

```
outdat <- local({
    res <- t(sapply(split(indat, indat[c("a", "b")]), function(subset) {
        c(xmean = weighted.mean(subset$x, subset$w),
          ymean = weighted.mean(subset$y, subset$w),
          yvar = cov.wt(subset["y"], subset$w)$cov)
    }))
    levs <- aggregate(w ~ a + b, data=indat, sum)
    cbind(levs, as.data.frame(res))
})
```

Thank god for `plyr`

Challenges for deploying R

- Quality assurance, or perception thereof
  - Core is excellent, but much of R's attraction is in extensibility, ie contributed packages
  - Can I be sure that the package I just downloaded does what it says? Is it doing *more* than it says?
  - Backward compatibility
    - Telling people to use the latest version is not always helpful
- No single point of contact
  - Who do I yell at if things go wrong?
  - How can I be sure everyone is using the same version?
- Unix roots make package development clunky on Windows
  - Process is more fragile because it assumes Unix conventions
  - Why must I download a set of third-party tools to compile code?
  - Difficult to integrate with Visual C++/Visual Studio
- Interfacing with languages other than Fortran, C, C++ not yet integrated into core

ANZ

Commercial R: an aside

- Many of these issues are fundamental in nature
- Third parties like Revolution Analytics can address them, without having to dilute R's focus (I am not a Revo R user)
  - Other R commercialisations existed but seem to have disappeared
  - Anyone remember S-Plus?
  - Challenge is not to negate R's drawcards
    - Low cost: important even for big companies
    - Community and ecosystem
      - Can I use Revo R with that package I got from CRAN?
      - If I use Revo R, can I/should I participate on R-Help, StackOverflow.com?
- Also why SAS, SPSS, etc can include interfaces to R without risking too much

ANZ

Good problems to have

- Sign of R's movement into the mainstream
- S-Plus now touts R compatibility, rather than R touting S compatibility
- Nobody cares that SHAZAM, GLIM, XLisp-Stat etc don't support XML, C# or Java

ANZ

Other resources

- SAS and R blog by Ken Kleinman and Nick Horton
  - Online support for their book, *SAS and R: Data Management, Statistical Analysis, and Graphics*
- Hadley Wickham's site
  - STAT480 covers Excel, R and SAS (the links to UCLA ATS files are broken, use `/stat/` instead of `/STAT/`)
- The DO Loop is the official SAS/IML blog
- inside-R, the Revolution Analytics community site
- R-bloggers: aggregated blogroll for R
- AnnMaria De Mars' blog on SAS and social science (contains pictures of naked mole rats)